QQQ – PureYr2 - Chapter 2 – Functions & Graphs (v1)

Total Marks: 22

(22 = Platinum, 20 = Gold, 18 = Silver, 16 = Bronze)

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1. Given

$f(x) = e^x$, $\quad x \in \mathbb{R}$

$g(x) = 3\ln x$, $\quad x > 0$, $x \in \mathbb{R}$

find an expression for $gf(x)$, simplifying your answer. **(2)**

2. The function f is defined by

$$f: x \to \frac{3x-5}{x+1}, x \in \mathbb{R}, x \neq -1$$

(a) Show that

$$ff(x) = \frac{x+a}{x-1}, x \in \mathbb{R}, x \neq \pm 1$$

where a is an integer to be found. **(4)**

(b) Find $f^{-1}(x)$ **(3)**

3. The function g is defined by

$$g: x \to x^2 - 3x, x \in \mathbb{R}, 0 \leq x \leq 5$$

Find the range of g . **(3)**

4.

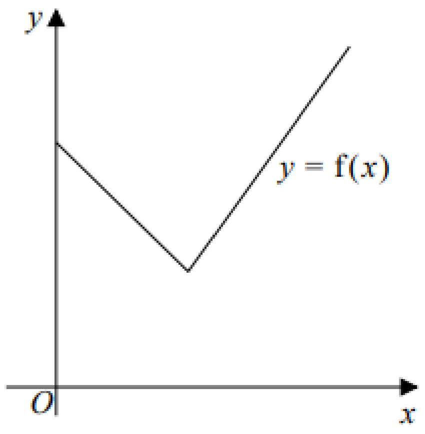

Figure 2

Figure 2 shows a sketch of part of the graph $y = f(x)$, where

$f(x) = 2|3 - x| + 5$, $x \geq 0$

(a) State the range of f . **(1)**

(b) Solve the equation $f(x) = \frac{1}{2}x + 30$ **(3)**

(c) Given that the equation $f(x) = k$, where k is a constant, has two distinct roots, state the set of possible values for k . **(2)**

5.

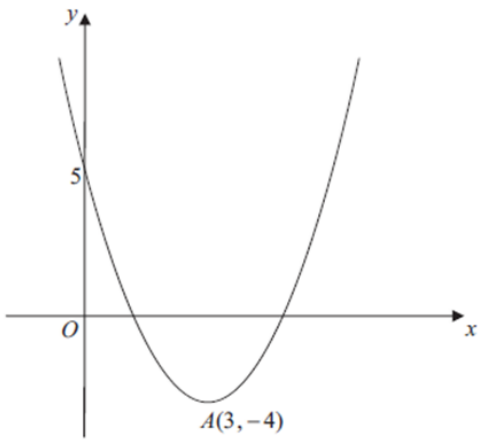

Figure 2

Figure 2 shows a sketch of the curve with the equation $y = f(x)$, $x \in \mathbb{R}$.

The curve has a turning point at $A(3, -4)$ and also passes through the point (0, 5).

(a) Write down the coordinates of the point to which A is transformed on the curve with equation $y = |f(x)|$ **(2)**

(b) Write down the coordinates of the point to which A is transformed on the curve with equation $y = 2f\left(\frac{1}{2}x\right)$ **(2)**

QQQ – PureYr2 - Chapter 2 – Functions & Graphs (v2)

Total Marks: 15

(15 = Platinum, 13 = Gold, 11 = Silver, 10 = Bronze)

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1. The function f is defined by

$f: x \to e^{2x} + k^2$, $x \in \mathbb{R}$, k is a positive constant.

The function g is defined by

$g: x \to \ ln\,(2x)$, $x > 0$

Find $fg(x)$, giving your answer in its simplest form. **(2)**

2. $g(x) = \frac{2x+5}{x-3}$ $x \geq 5$

(a) Find $g^{-1}(x)$, stating its domain. **(3)**

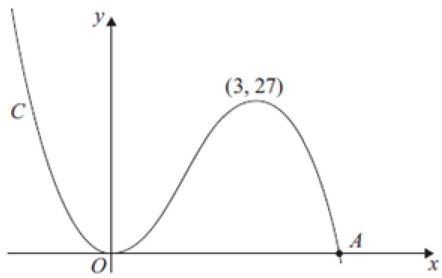
(b) State the range of g . **(1)**

3. By a suitable sketch of $y = |4x - 3|$ or otherwise, find the complete set of values of x for which

(a) $|4x - 3| > 2 - 2x$ **(4)**

(b) $|4x - 3| > \frac{3}{2} - 2x$ **(2)**

4. The figure shows a sketch of the curve with equation $y = f(x)$, where $f(x) = x^2(9 - 2x)$. There is a minimum at the origin, a maximum at the point $(3,27)$ and C cuts the x -axis at the point A .



Write down the coordinates of the maximum point of the line with equation $y = f(3x)$. **(3)**

Review question R5595

Question

(a) The functions f and g are defined by

$$f : x \mapsto e^x, \qquad x \in \mathbb{R},$$
$$g : x \mapsto x - 1, \qquad x \in \mathbb{R}.$$

Sketch in a single diagram the graphs of f and the composite functions gf and fg, labelling each graph clearly.

State briefly the relationship

(i) between the graphs of f and gf,

(ii) between the graphs of f and fg.

(b) Express $(1-x)(x-3)$ in the form $a-(x-b)^2$, where a and b are constants. State the coordinates of the maximum point on the graph of $y=(1-x)(x-3)$, and state what symmetry the graph possesses.

Sketch the graph of $y = e^{(1-x)(x-3)}$.

UCLES A level Mathematics C 1, QP 9205/1 and 9206/1, 1986, Q13

Compose!

Problem

Part of the definition of a function is the specification of its domain. The choice of domain may affect the range of the function.

✱

What is the largest possible domain for each of the functions A to D and what is the corresponding range?

$$A(x) = x^2 - 2 \qquad B(x) = 2x + 4$$

$$C(x) = \frac{1}{x} \qquad D(x) = \sqrt{x+2} - 2$$

The functions above have been composed in some way to make the following new functions:

(1) $f(x) = \dfrac{2}{x} + 4$

(2) $f(x) = 4x^2 + 16x + 14$

(3) $f(x) = \dfrac{1}{2x^2}$

(4) $f(x) = x - 2$

(5) $f(x) = \dfrac{1}{4x + 12}$

(6) $f(x) = x$

✱

Can you work out which functions have been composed and in what order to make the new functions?

What are the domain and range for each composition of functions above?

☞ Note that the *composition* might only be defined on a smaller domain than is possible for the function $f(x)$.