Straight line graphs – Perpendicular lines

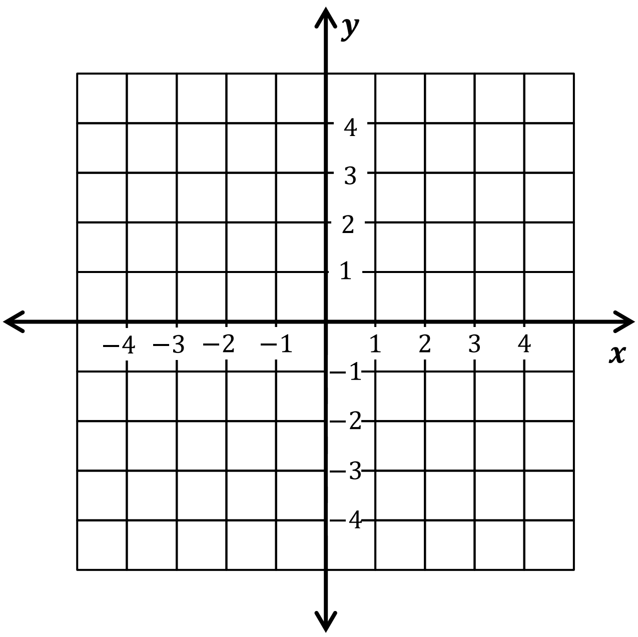
Relevant formulas:

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \dfrac{change\ in\ y}{change\ in\ x}$ $\quad ax + by = c$ $\quad y = mx + c$ $\quad$ *parallel lines have the same gradient*

1. a) Complete the table of values below for $y_1 = 2x + 1$

$\boldsymbol{x}$	$\boldsymbol{-3}$	$\boldsymbol{-2}$	$\boldsymbol{-1}$	$\boldsymbol{0}$	$\boldsymbol{1}$	$\boldsymbol{2}$
$\boldsymbol{y_1}$				1	3	

b) On the grid below, draw the graph for $y_1 = 2x + 1$ by plotting the points from the table.



c) Write down the **gradient** for the line $y_1 = 2x + 1$

$m_1 =$ ____

d) Complete the table of values below for $y_2 = -\frac{1}{2}x - 2$

$\boldsymbol{x}$	$\boldsymbol{-4}$	$\boldsymbol{-2}$	$\boldsymbol{0}$	$\boldsymbol{2}$	$\boldsymbol{4}$
$\boldsymbol{y}$			-2	-3	

e) Draw the graph for $y_2 = -\frac{1}{2}x - 2$ on the same grid as b) above.

f) Write down the **gradient** for the line $y_2 = -\frac{1}{2}x - 2$

$m_2 =$ ____

f) Are these lines parallel or perpendicular? Yes / No

Two straight lines with equations $y = 2x + 1$ and $y = -\frac{1}{2}x - 2$ are perpendicular when: _____ × _____ $= -1$

For example, 2 becomes $-\frac{1}{2}$

$\underline{\boldsymbol{m_1}} \times \underline{\boldsymbol{m_2}} = -1$

Gradient of the perpendicular = Flip the 2 and change the sign !!!

2) i) Write down the gradient and the y-intercept of the following straight lines:

a) $y = -3x + 2$ $m =$ ____ $c =$ ____

b) $4x - 2y = -5$ $m =$ ____ $c =$ ____

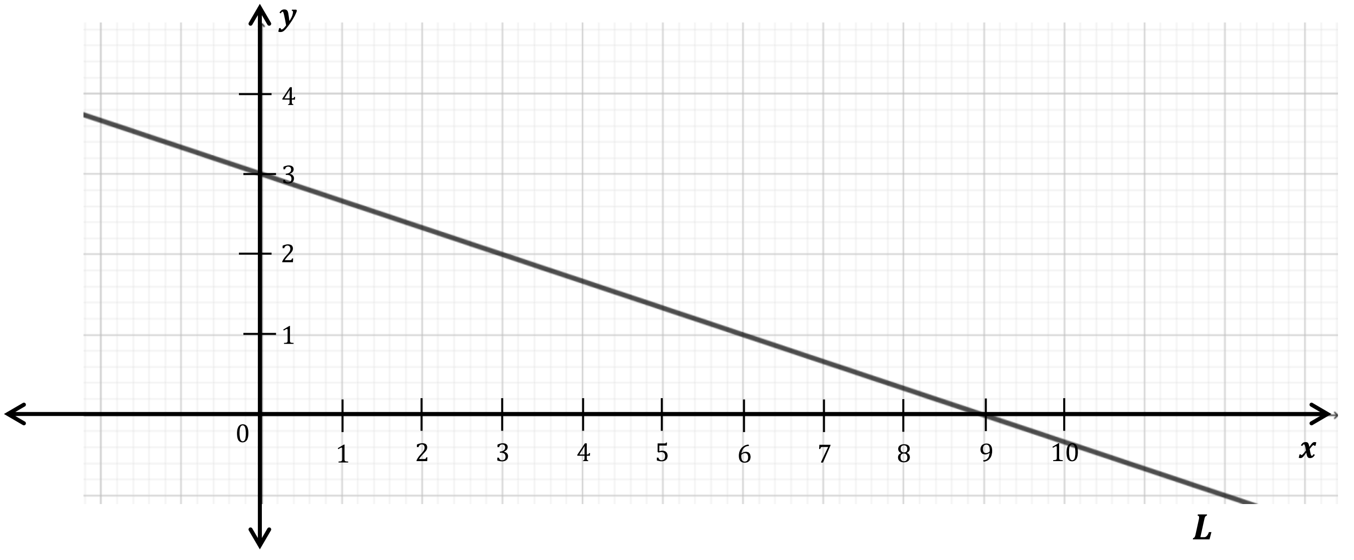
c) $3y - x = 7$ $m =$ ____ $c =$ ____

ii) Which of the above lines are perpendicular to each other? Show all your working.

3) A straight line L, **passes through** the point with coordinates $(4{,}7)$ and is **perpendicular** to the line with equation $y = -4x + 1$

Find an equation of the straight line L.

4) i) **Find the equation of line L** below. You must show all your working.



ii) Find the **gradient** and the **y-intercept** of line **P** which is **perpendicular** to line L above and **passes through** (1, 2). Hence write down the equation of the straight line P in the form $y = mx + c$

iii) On the same grid above, sketch the graph of the perpendicular line P.

Straight lines

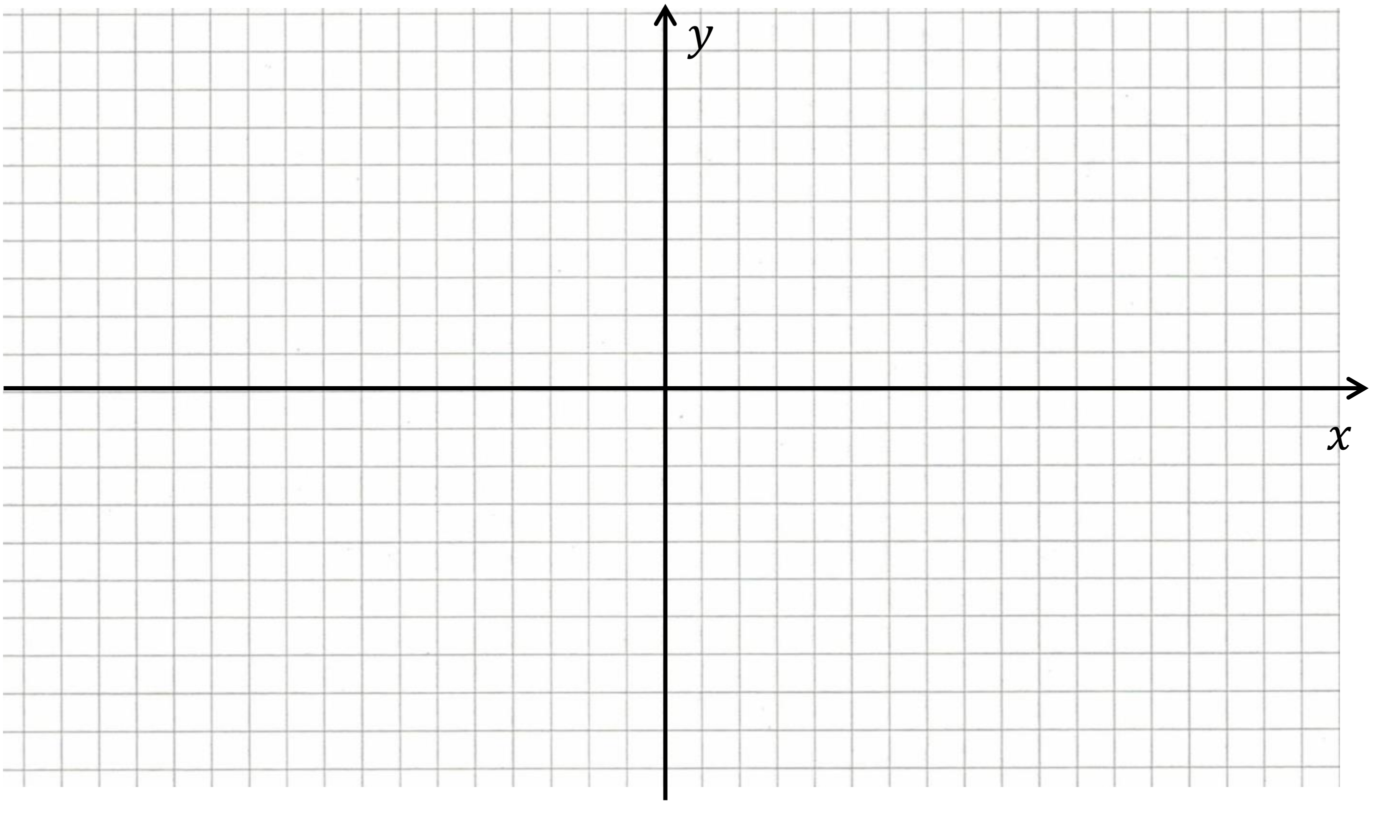
1. a) Show that the straight lines $L_1: \mathrm{y} - 2\mathrm{x} + 3 = 0$ and $L_2: 2y + x + 2 = 0$ are perpendicular.

   b) Find the equation of line L_3 which is perpendicular to L_2 and passes through the point with coordinates $(-2,3)$.

   c) *"Lines L_1 and L_3 must be parallel to each other"*. Comment on the validity of this statement.

   d) Find the point of intersection of lines L_1 and L_2.

   e) Find the shortest distance between lines L_1 and L_3



2. The points with coordinates $\mathrm{A}(4,6)$ and $\mathrm{B}(2,0)$ lies on the same straight line L_1.

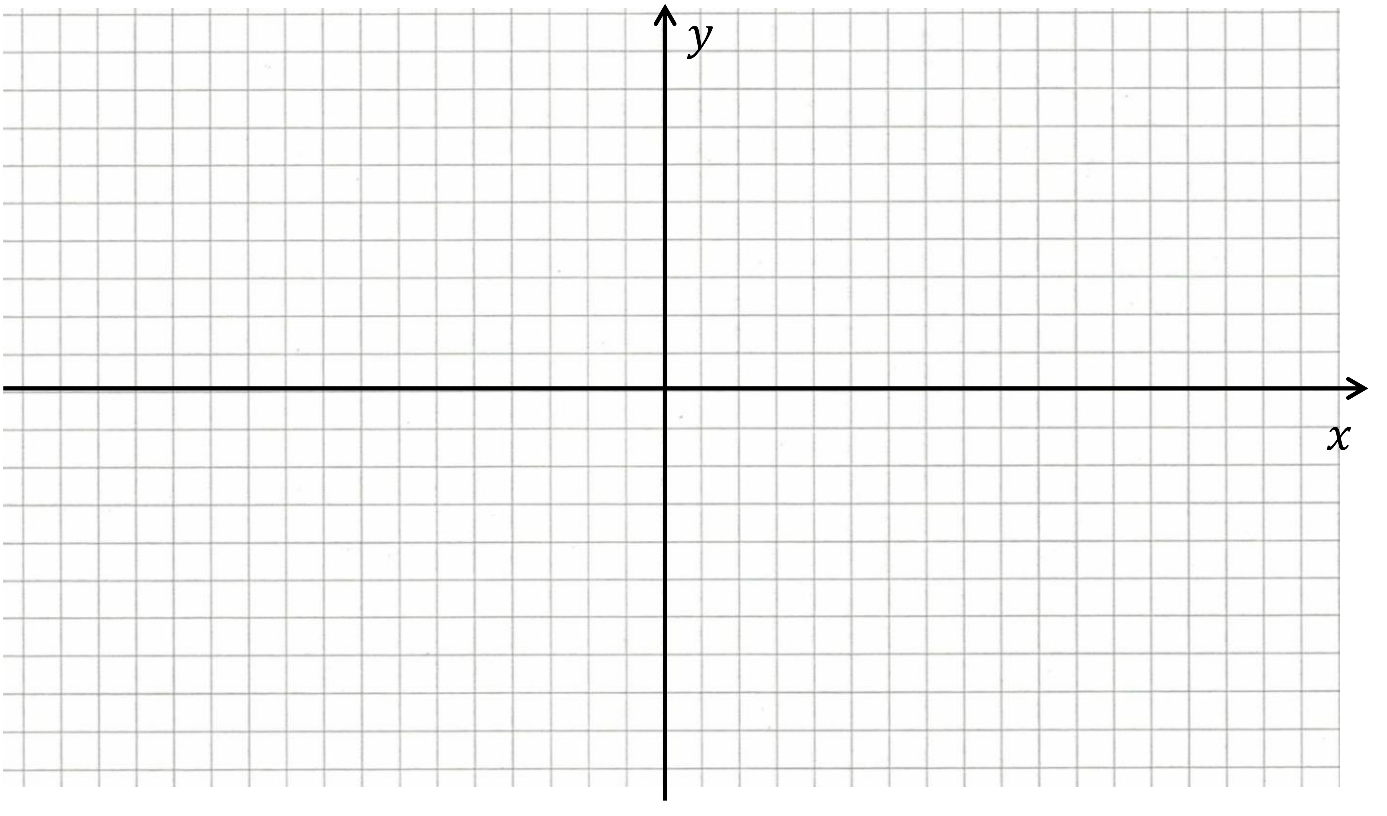
a) Find the equation of L_1. Give your answer in the form $ax + by + c = 0$

b) Line L_2 is perpendicular to L_1 passing through point B. Find the equation of line L_2. Give your answer in the form $ay = bx + c$ where a, b and c are constants to be found.

c) Work out the y intercept of line L_2. Label this point C.

d) Sketch lines L_1 and L_2 on the diagram below. Indicate points A, B and C.

e) Find the area of triangle ABC. Give your answer correct in 2 d.p.



3. In the triangle ABC the equations of the two sides are

$$L_1:\ 2y = x + 2 \text{ and } L_2:\ y = 2x - 2$$

Given that point A has coordinates $A(4,3)$ and that angle ABC is $90°$

i) Find the distance between the x intercept of line L_1 and the y intercept of line L_2.

ii) Draw the triangle on a grid, indicating all the relevant line-graph and points A, B and C

ii) Find the equation of the third line forming the triangle

***www.undergroundmathematics, R 5661**

Name: ______________________________

GCSE (1 – 9)

Solving Simultaneous Equations Graphically

Instructions

• Use **black** ink or ball-point pen.
• Answer all Questions.
• Answer the Questions in the spaces provided
– *there may be more space than you need.*
• Diagrams are **NOT** accurately drawn, unless otherwise indicated.
• You must **show all your working out.**

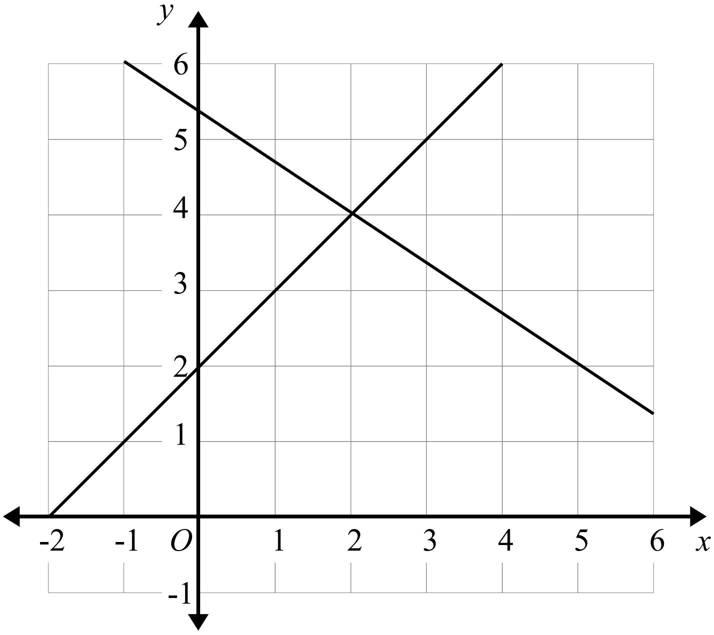
Information

• The marks for each Question are shown in brackets
– *use this as a guide as to how much time to spend on each Question.*

Advice

• Read each Question carefully before you start to answer it.
• Keep an eye on the time.
• Try to answer every Question.
• Check your answers if you have time at the end

1 The graphs of the straight lines with equations $y = x + 2$ and $2x + 3y = 16$ have been drawn on the grid.



Use the graphs to solve the simultaneous equations

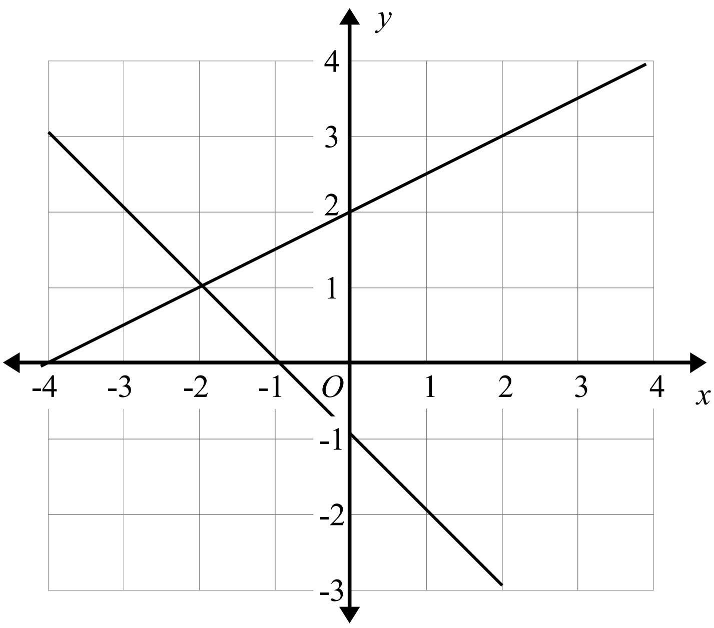
$$y = x + 2$$
$$2x + 3y = 16$$

....

(Total for Question 1 is 2 marks)

---

2 The graphs of the straight lines with equations $2y - x = 4$ and $x + y = -1$ have been drawn on the grid.



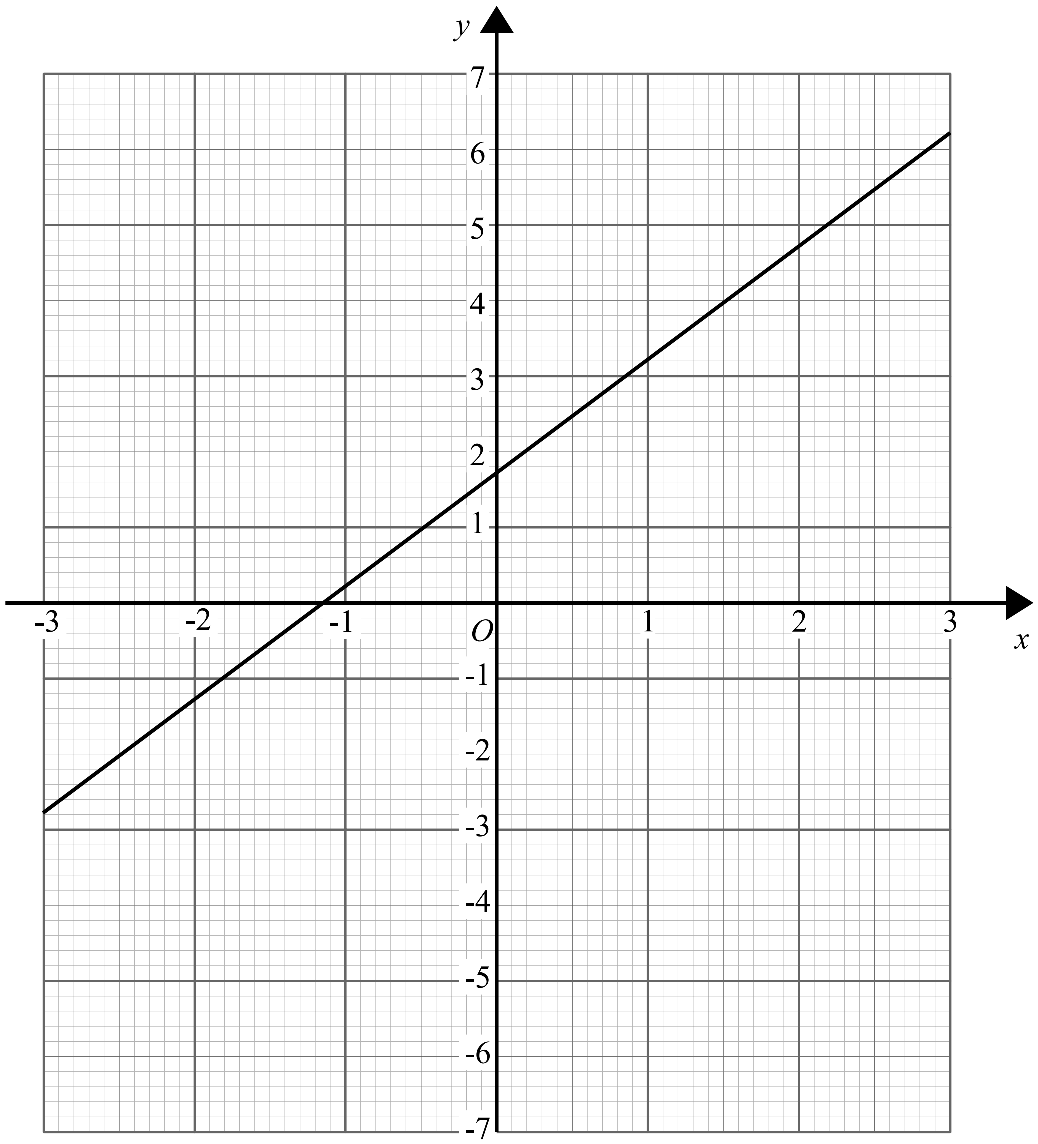
Use the graphs to solve the simultaneous equations

$$2y - x = 4$$
$$x + y = -1$$

....

(Total for Question 2 is 2 marks)

3 The graph of $4y - 6x = 7$ is drawn on the grid.



(a) On the grid , draw the graph of $y = -2x$

(2)

(b) Use the graphs to solve the simultaneous equations

$$4y - 6x = 7$$
$$y = -2x$$

$x =$ ....

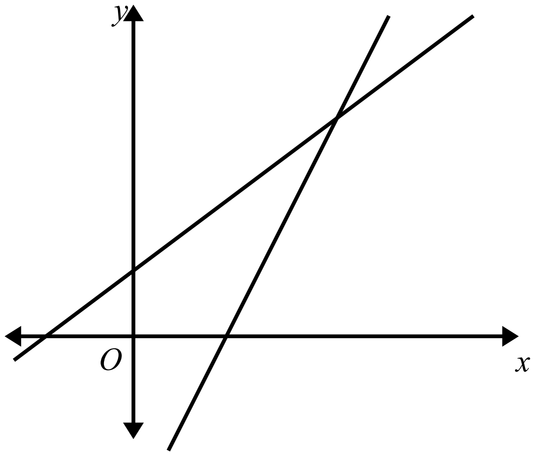
$y =$ ....

(2)

(Total for Question 3 is 4 marks)

4 The diagram shows two straight lines.
The equation of the lines are $y = 4x - 5$ and $y = 2x + 1$

Work out the coordinates of the point where the line intersect.

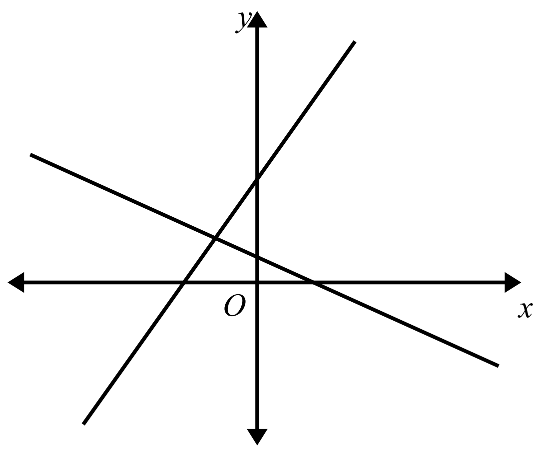


....

(Total for Question 4 is 3 marks)

5 The diagram shows two straight lines.
The equation of the lines are $y = 2x + 3$ and $y = -\frac{2}{3}x + 1$

Work out the coordinates of the point where the line intersect.



....

(Total for Question 5 is 3 marks)

<u>Straight Line Graphs</u>

When the equation of a straight line is written in the form

$$y = mx + c$$

m *is the* ***gradient*** *and* ***c*** *is the* ***y-intercept*** *(where the line crosses the y-axis).*

1. Complete the following table

e.g. $y = 5x + 2$

In this case $m = +5$ and $c = +2$

i.e. the gradient is 5 and the straight line will cross the axis at the point (0,2).

Equation	Gradient (m)	y-intercept (c)
$y = 5x + 2$	5	2
$y = 4x + 7$		
$y = \frac{1}{2}x - 4$		
$y = 3x - 2$		
$y = 7x$		
$y = \frac{3}{4}x + \frac{1}{2}$		
$y = -4x + 7$		
$y = -3 - 7x$		

2. Complete the table using the information given

Gradient (m)	y-intercept (c)	Equation ($y = mx + c$)
2	3	
2	−5	
0	2	
4	0	
5	−4	
−4	9	
−3	−2	
¾	−½	

Sometimes the equation of a line is not given in the form $y = mx + c$. In these cases we need to rearrange our equations until they take the form $y = mx + c$.

.e.g.	$x + y = 2$
Subtract x from both sides	$x + y - x = 2 - x$
Then rearrange	$y = -x + 2$
Now we can see that	
$m = -1$ *and* $c = 2$	

3. Complete the following table

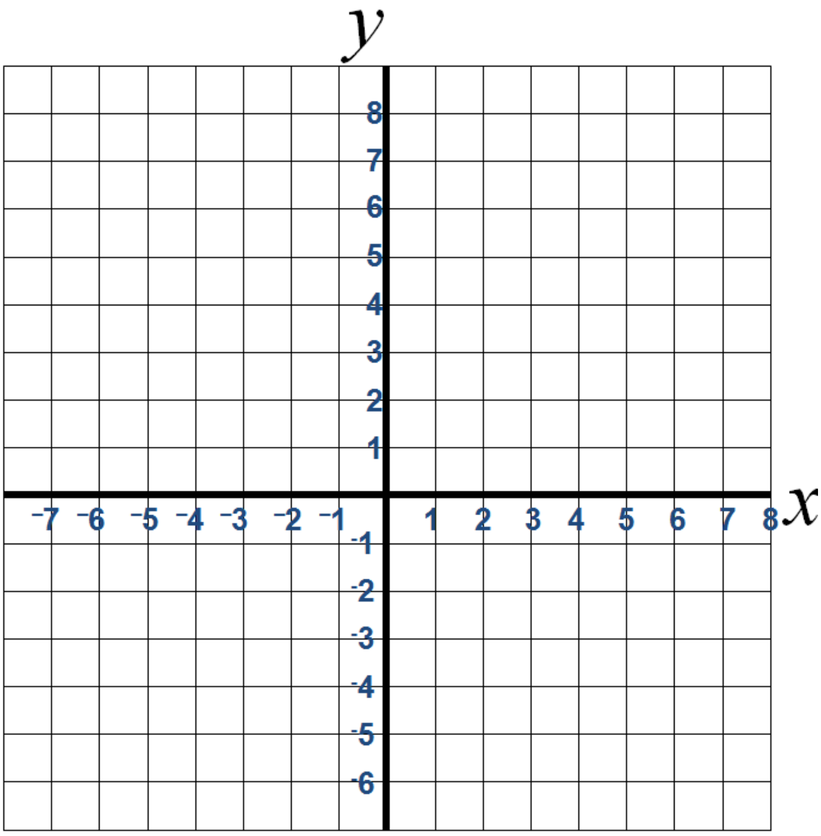
These are a bit tricky. Be Careful!

Equation	Rearranged equation in the form y = mx + c	Gradient (m)	y-intercept (c)
$x + y = 2$	$y = -x + 2$	−1	2
$x + y = -4$			
$y - 3x = 1$			
$y = 3(x + 5)$			
$y = -2(x - ½)$			
$x - y = 0$			
$3y = 6x - 9$			
$3x + 2y = 4$			
$3y = (x - 4)$			
$4x - 5y = -5$			
$\frac{x}{3} + \frac{y}{4} = \frac{5}{6}$			

Straight line graphs

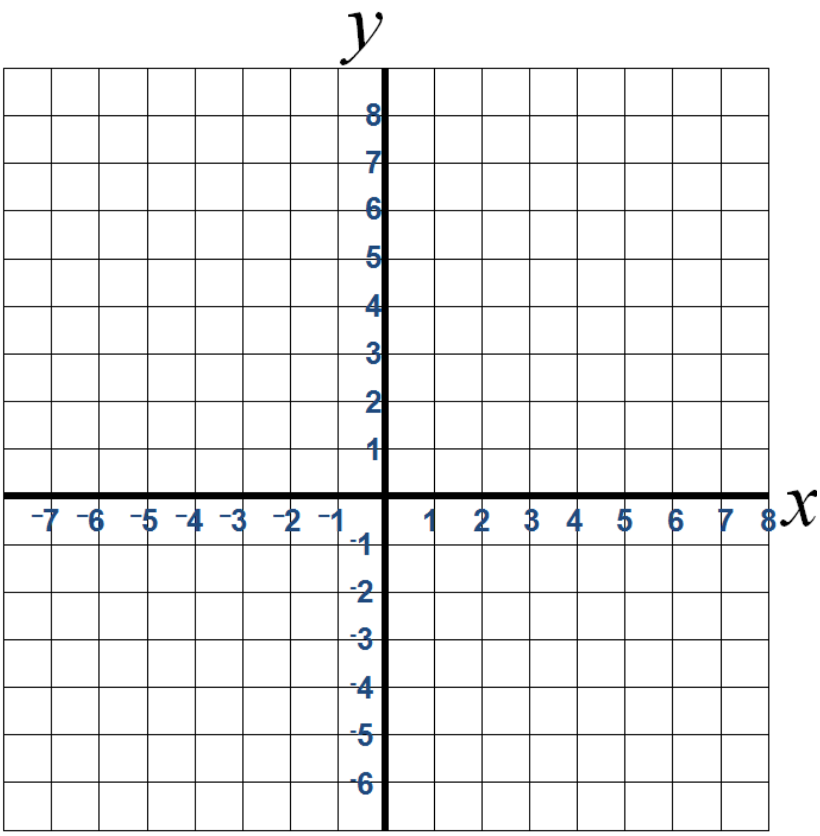
PLOT

1) $x + y = 4$
2) $2x + y = 2$
3) $x + 2y = 2$
4) $x + 3y = 6$
5) $2x + 5y = 10$



PLOT

6) $x - y = 3$
7) $2x - y = 2$
8) $2x - 3y = 6$
9) $x + 2y = 1$
10) $2x - y = 3$



Write down the gradient and y-intercept of each line.

(a) $y = 2x + 3$

(b) $y = 4x - 2$

(c) $y = \frac{1}{2}x + 1$

(d) $y = \frac{2}{3}x - 4$

(e) $y = 4(x + 2)$

(f) $y = 3(x - 7)$

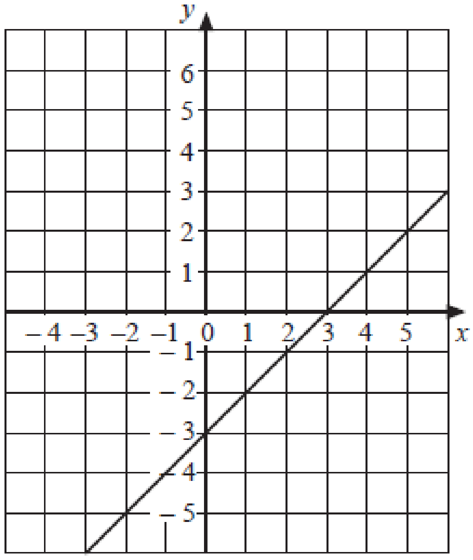
(g) $y = \frac{x + 5}{2}$

(h) $y = \frac{x - 10}{4}$

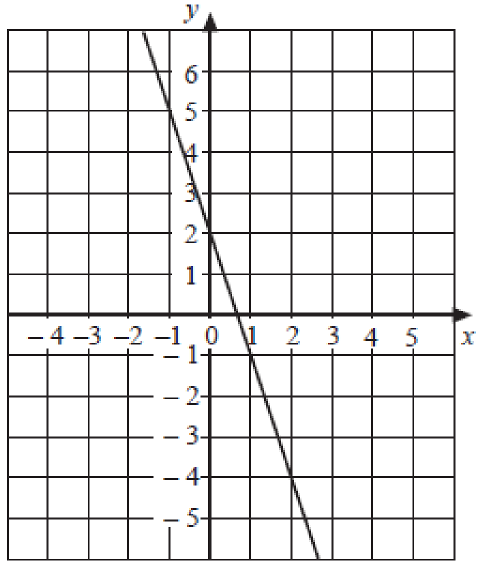
Straight line graphs

1. Find the equations of the lines

(a)



(b)



2. What are the gradients and y intercepts of the following equations?

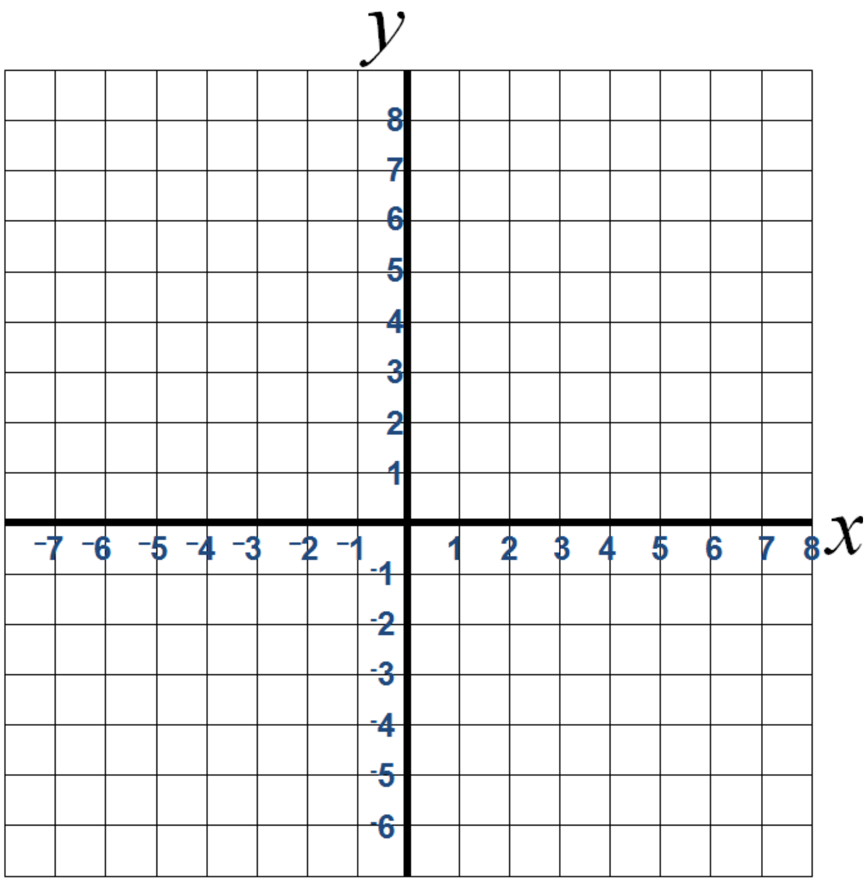
a) $\boldsymbol{y} = -\frac{1}{3}\boldsymbol{x} + 5$

b) $\boldsymbol{y} = \frac{\boldsymbol{x}}{4} + 3$

3. Plot the following lines from -5 to 5:

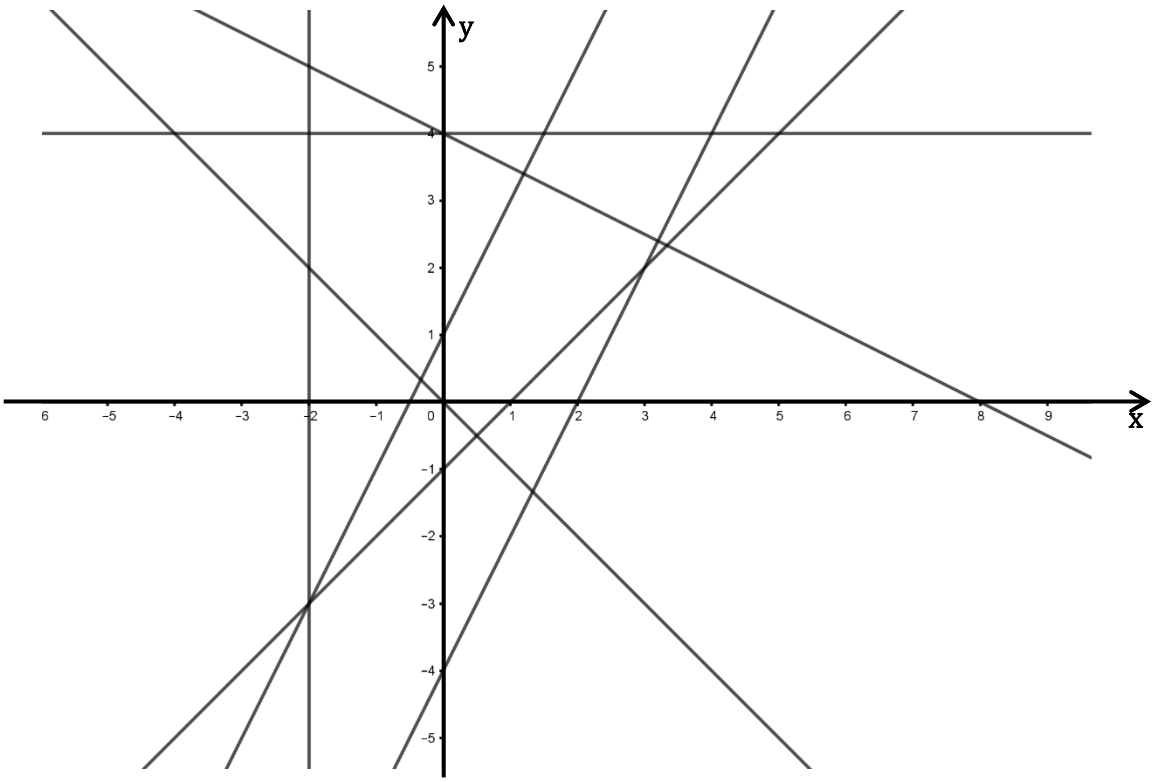
a) $\boldsymbol{y} = -\boldsymbol{x} - 1$

x										
y										



b) $2\boldsymbol{x} + 3\boldsymbol{y} = 6$

Equations of Straight Lines Challenge



1. i) Find the x-intercept and the y-intercept for each line.

   ii) Hence find the equation of each line in the form $y = mx + c$

   iii) Identify the pairs of lines which are parallel to each other.

   iv) Identify the pairs of lines which are perpendicular to each other.

2. Use the information below to find the equation of each line in the form $y = mx + c$

   a) gradient = 3, y-intercept $(0, -5)$

   b) gradient = -1, passes through $(3,8)$

   c) gradient = 2, x-intercept $(-3,0)$

   d) passes through $(3, -4)$ and $(5,0)$

   e) passes through $(-7, -6)$ and $(-3,4)$

3. Does the line $y = 3x - 7$ pass through the point $(6,11)$? Explain your answer.

4. Does the line $y = 18 - 5x$ pass through the point $(4, -4)$? Explain your answer.

5. Find the gradient, y-intercept and x-intercept for each graph:

   a) $y = 4x - 8$

   b) $4y = 12x + 2$

   c) $5x + 2y = 8$

   d) $3x - 6y + 9 = 0$

6. Decide whether each statement is “always true”, “always false” or “sometimes true” explaining your decision in each case:

A: If two lines have different gradients they must intersect.

B: If a line intersects the y-axis at (0,-3) it intersects the *positive* x axis.

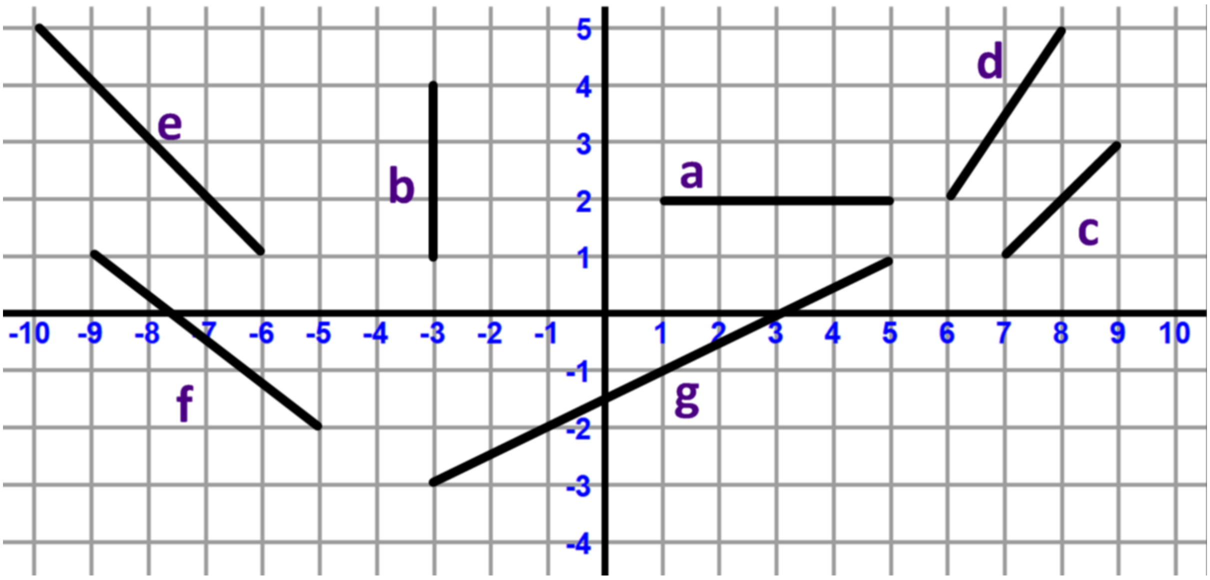
C: If a line has a gradient of 1 it makes an angle of 45^0 with the x-axis.

D: If a line cuts the positive x-axis and the positive y-axis it has a negative gradient.

E: The line x=4 has a gradient of zero.

Midpoint of a Line

1. Find the midpoint of the following line segments.



2. Calculate the midpoint of the line segments joining the two points below.

a) (3, 6) and (5, 10)

b) (0, 1) and (4, 3)

c) (-3, 2) and (3, 2)

d) (-8, -7) and (4, 9)

e) (-7, 10) and (0, -5)

f) (12, 0) and (0, -0.5)

3. The midpoint of the line segment AB is at (3.5, 2.5). The coordinates of A are (2, 5). Work out the coordinates of B.

4. The coordinates (2, -1), (12, -1), (12, 9), (2, 9) are joined up to form a square. Find the centre of the square. Find the midpoints of the diagonals of the square. What do you notice?

5. Draw a 2D square in your book. Find the midpoints of each of the sides. Join up the midpoints to form another shape. Repeat the process for this new shape. What do you notice?