

# C2 DIFFERENTIATION

## Worksheet A

1 In each case, find any values of  $x$  for which  $\frac{dy}{dx} = 0$ .

**a**  $y = x^2 + 6x$       **b**  $y = 4x^2 + 2x + 1$       **c**  $y = x^3 - 12x$       **d**  $y = 4 + 9x^2 - x^3$   
**e**  $y = x^3 - 5x^2 + 3x$       **f**  $y = x + \frac{9}{x}$       **g**  $y = (x^2 + 3)(x - 3)$       **h**  $y = x^{\frac{1}{2}} - 2x$

2 Find the set of values of  $x$  for which  $f(x)$  is increasing when

**a**  $f(x) \equiv 2x^2 + 2x + 1$       **b**  $f(x) \equiv 3x^2 - 2x^3$       **c**  $f(x) \equiv 3x^3 - x - 7$   
**d**  $f(x) \equiv x^3 + 6x^2 - 15x + 8$       **e**  $f(x) \equiv x(x - 6)^2$       **f**  $f(x) \equiv 2x + \frac{8}{x}$

3 Find the set of values of  $x$  for which  $f(x)$  is decreasing when

**a**  $f(x) \equiv x^3 + 2x^2 + 1$       **b**  $f(x) \equiv 5 + 27x - x^3$       **c**  $f(x) \equiv (x^2 - 2)(2x - 1)$

4  $f(x) \equiv x^3 + kx^2 + 3.$

Given that  $(x + 1)$  is a factor of  $f(x)$ ,

- a** find the value of the constant  $k$ ,  
**b** find the set of values of  $x$  for which  $f(x)$  is increasing.

5 Find the coordinates of any stationary points on each curve.

**a**  $y = x^2 + 2x$       **b**  $y = 5x^2 - 4x + 1$       **c**  $y = x^3 - 3x + 4$   
**d**  $y = 4x^3 + 3x^2 + 2$       **e**  $y = 2x + 3 + \frac{8}{x}$       **f**  $y = x^3 - 9x^2 - 21x + 11$   
**g**  $y = \frac{1}{x} - 4x^2$       **h**  $y = 2x^{\frac{3}{2}} - 6x$       **i**  $y = 9x^{\frac{2}{3}} - 2x + 5$

6 Find the coordinates of any stationary points on each curve. By evaluating  $\frac{d^2y}{dx^2}$  at each stationary point, determine whether it is a maximum or minimum point.

**a**  $y = 5 + 4x - x^2$       **b**  $y = x^3 - 3x$       **c**  $y = x^3 + 9x^2 - 8$   
**d**  $y = x^3 - 6x^2 - 36x + 15$       **e**  $y = x^4 - 8x^2 - 2$       **f**  $y = 9x + \frac{4}{x}$   
**g**  $y = x - 6x^{\frac{1}{2}}$       **h**  $y = 3 - 8x + 7x^2 - 2x^3$       **i**  $y = \frac{x^4 + 16}{2x^2}$

7 Find the coordinates of any stationary points on each curve and determine whether each stationary point is a maximum, minimum or point of inflexion.

**a**  $y = x^2 - x^3$       **b**  $y = x^3 + 3x^2 + 3x$       **c**  $y = x^4 - 2$   
**d**  $y = 4 - 12x + 6x^2 - x^3$       **e**  $y = x^2 + \frac{16}{x}$       **f**  $y = x^4 + 4x^3 - 1$

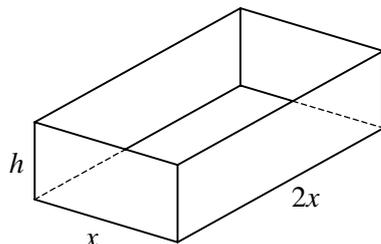
8 Sketch each of the following curves showing the coordinates of any turning points.

**a**  $y = x^3 + 3x^2$       **b**  $y = x + \frac{1}{x}$       **c**  $y = x^3 - 3x^2 + 3x - 1$   
**d**  $y = 3x - 4x^{\frac{1}{2}}$       **e**  $y = x^3 + 4x^2 - 3x - 5$       **f**  $y = (x^2 - 2)(x^2 - 6)$

## C2 DIFFERENTIATION

## Worksheet B

1



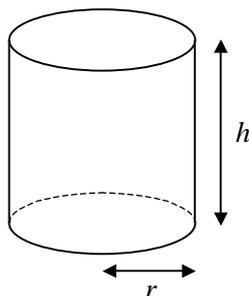
The diagram shows a baking tin in the shape of an open-topped cuboid made from thin metal sheet. The base of the tin measures  $x$  cm by  $2x$  cm, the height of the tin is  $h$  cm and the volume of the tin is  $4000 \text{ cm}^3$ .

- a Find an expression for  $h$  in terms of  $x$ .
- b Show that the area of metal sheet used to make the tin,  $A \text{ cm}^2$ , is given by

$$A = 2x^2 + \frac{12000}{x}.$$

- c Use differentiation to find the value of  $x$  for which  $A$  is a minimum.
- d Find the minimum value of  $A$ .
- e Show that your value of  $A$  is a minimum.

2



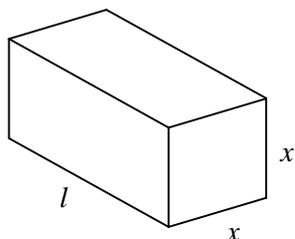
The diagram shows a closed plastic cylinder used for making compost. The radius of the base and the height of the cylinder are  $r$  cm and  $h$  cm respectively and the surface area of the cylinder is  $30\,000 \text{ cm}^2$ .

- a Show that the volume of the cylinder,  $V \text{ cm}^3$ , is given by

$$V = 15\,000r - \pi r^3.$$

- b Find the maximum volume of the cylinder and show that your value is a maximum.

3



The diagram shows a square prism of length  $l$  cm and cross-section  $x$  cm by  $x$  cm. Given that the surface area of the prism is  $k \text{ cm}^2$ , where  $k$  is a constant,

- a show that  $l = \frac{k - 2x^2}{4x}$ ,
- b use calculus to prove that the maximum volume of the prism occurs when it is a cube.

## C2 DIFFERENTIATION

## Worksheet C

1  $f(x) \equiv 2x^3 + 5x^2 - 1.$

- a Find  $f'(x)$ .  
 b Find the set of values of  $x$  for which  $f(x)$  is increasing.

2 The curve  $C$  has the equation  $y = x^3 - x^2 + 2x - 4.$

- a Find an equation of the tangent to  $C$  at the point  $(1, -2)$ . Give your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.  
 b Prove that the curve  $C$  has no stationary points.

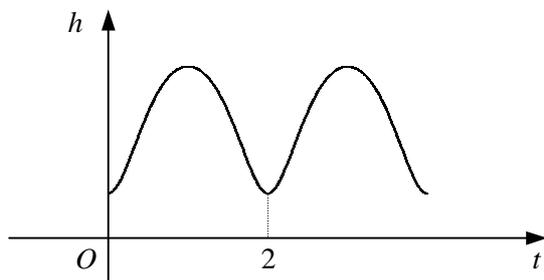
3 A curve has the equation  $y = \sqrt{x} + \frac{4}{x}.$

- a Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ .  
 b Find the coordinates of the stationary point of the curve and determine its nature.

4  $f(x) \equiv x^3 + 6x^2 + 9x.$

- a Find the coordinates of the points where the curve  $y = f(x)$  meets the  $x$ -axis.  
 b Find the set of values of  $x$  for which  $f(x)$  is decreasing.  
 c Sketch the curve  $y = f(x)$ , showing the coordinates of any stationary points.

5



The graph shows the height,  $h$  cm, of the letters on a website advert  $t$  seconds after the advert appears on the screen.

For  $t$  in the interval  $0 \leq t \leq 2$ ,  $h$  is given by the equation

$$h = 2t^4 - 8t^3 + 8t^2 + 1.$$

For larger values of  $t$ , the variation of  $h$  over this interval is repeated every 2 seconds.

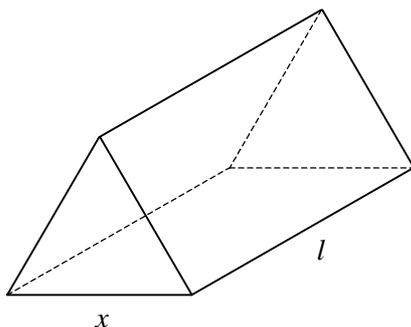
- a Find  $\frac{dh}{dt}$  for  $t$  in the interval  $0 \leq t \leq 2$ .  
 b Find the rate at which the height of the letters is increasing when  $t = 0.25$   
 c Find the maximum height of the letters.

6 The curve  $C$  has the equation  $y = x^3 + 3kx^2 - 9k^2x$ , where  $k$  is a non-zero constant.

- a Show that  $C$  is stationary when  

$$x^2 + 2kx - 3k^2 = 0.$$
  
 b Hence, show that  $C$  is stationary at the point with coordinates  $(k, -5k^3)$ .  
 c Find, in terms of  $k$ , the coordinates of the other stationary point on  $C$ .

7



The diagram shows a solid triangular prism. The cross-section of the prism is an equilateral triangle of side  $x$  cm and the length of the prism is  $l$  cm.

Given that the volume of the prism is  $250 \text{ cm}^3$ ,

- find an expression for  $l$  in terms of  $x$ ,
- show that the surface area of the prism,  $A \text{ cm}^2$ , is given by

$$A = \frac{\sqrt{3}}{2} \left( x^2 + \frac{2000}{x} \right).$$

Given that  $x$  can vary,

- find the value of  $x$  for which  $A$  is a minimum,
- find the minimum value of  $A$  in the form  $k\sqrt{3}$ ,
- justify that the value you have found is a minimum.

8

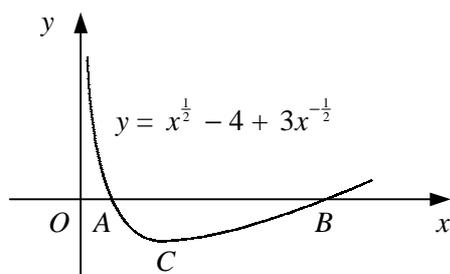
$$f(x) \equiv x^3 + 4x^2 + kx + 1.$$

- Find the set of values of the constant  $k$  for which the curve  $y = f(x)$  has two stationary points.

Given that  $k = -3$ ,

- find the coordinates of the stationary points of the curve  $y = f(x)$ .

9



The diagram shows the curve with equation  $y = x^{\frac{1}{2}} - 4 + 3x^{-\frac{1}{2}}$ . The curve crosses the  $x$ -axis at the points  $A$  and  $B$  and has a minimum point at  $C$ .

- Find the coordinates of  $A$  and  $B$ .
- Find the coordinates of  $C$ , giving its  $y$ -coordinate in the form  $a\sqrt{3} + b$ , where  $a$  and  $b$  are integers.

10

$$f(x) = x^3 - 3x^2 + 4.$$

- Show that  $(x + 1)$  is a factor of  $f(x)$ .
- Fully factorise  $f(x)$ .
- Hence state, with a reason, the coordinates of one of the turning points of the curve  $y = f(x)$ .
- Using differentiation, find the coordinates of the other turning point of the curve  $y = f(x)$ .

## C2 DIFFERENTIATION

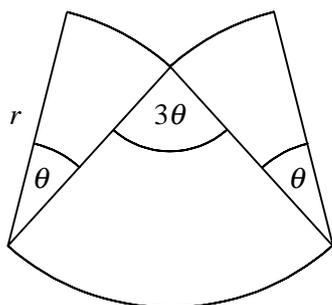
## Worksheet D

- 1  $f(x) \equiv 7 + 24x + 3x^2 - x^3$ .
- a Find  $f'(x)$ . (2)
- b Find the set of values of  $x$  for which  $f(x)$  is increasing. (4)

- 2 The curve with equation  $y = x^3 + ax^2 - 24x + b$ , where  $a$  and  $b$  are constants, passes through the point  $P(-2, 30)$ .
- a Show that  $4a + b + 10 = 0$ . (2)
- Given also that  $P$  is a stationary point of the curve,
- b find the values of  $a$  and  $b$ , (4)
- c find the coordinates of the other stationary point on the curve. (3)

- 3  $f(x) \equiv x^2 + \frac{16}{x}$ ,  $x \neq 0$ .
- a Find  $f'(x)$ . (2)
- b Find the coordinates of the stationary point of the curve  $y = f(x)$  and determine its nature. (6)

4



The diagram shows a design to be used on a new brand of cat-food. The design consists of three circular sectors, each of radius  $r$  cm. The angle of two of the sectors is  $\theta$  radians and the angle of the third sector is  $3\theta$  radians as shown.

Given that the area of the design is  $25 \text{ cm}^2$ ,

- a show that  $\theta = \frac{10}{r^2}$ , (3)
- b find the perimeter of the design,  $P$  cm, in terms of  $r$ . (3)
- Given that  $r$  can vary,
- c find the value of  $r$  for which  $P$  takes its minimum value, (4)
- d find the minimum value of  $P$ , (1)
- e justify that the value you have found is a minimum. (2)

- 5 The curve  $C$  has the equation

$$y = 2x - x^{\frac{3}{2}}, \quad x \geq 0.$$

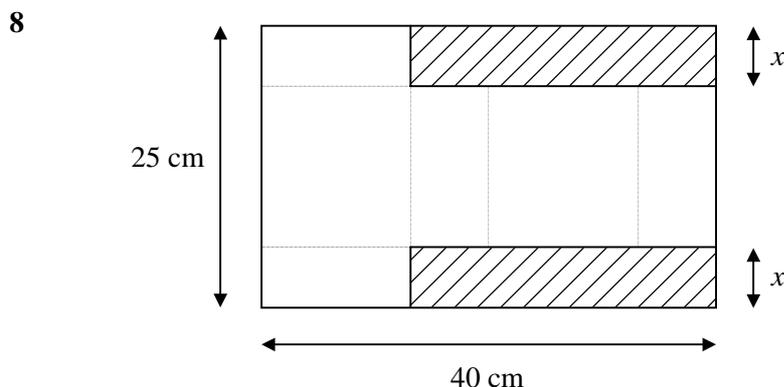
- a Find the coordinates of any points where  $C$  meets the  $x$ -axis. (3)
- b Find the  $x$ -coordinate of the stationary point on  $C$  and determine whether it is a maximum or a minimum point. (6)
- c Sketch the curve  $C$ . (2)

## C2 DIFFERENTIATION

## Worksheet D continued

- 6 The curve  $y = x^3 - 3x + 1$  is stationary at the points  $P$  and  $Q$ .
- a Find the coordinates of the points  $P$  and  $Q$ . (5)
- b Find the length of  $PQ$  in the form  $k\sqrt{5}$ . (3)

- 7  $f(x) \equiv 2x - 5 + \frac{2}{x}, x \neq 0$ .
- a Solve the equation  $f(x) = 0$ . (4)
- b Solve the equation  $f'(x) = 0$ . (4)
- c Sketch the curve  $y = f(x)$ , showing the coordinates of any turning points and of any points where the curve crosses the coordinate axes. (3)



Two identical rectangles of width  $x$  cm are removed from a rectangular piece of card measuring 25 cm by 40 cm as shown in the diagram above. The remaining card is the net of a cuboid of height  $x$  cm.

- a Find expressions in terms of  $x$  for the length and width of the base of the cuboid formed from the net. (3)
- b Show that the volume of the cuboid is  $(2x^3 - 65x^2 + 500x)$  cm<sup>3</sup>. (2)
- c Find the value of  $x$  for which the volume of the cuboid is a maximum. (5)
- d Find the maximum volume of the cuboid and show that it is a maximum. (3)
- 9 a Find the coordinates of the stationary points on the curve
- $$y = 2 + 9x + 3x^2 - x^3. \quad (6)$$
- b Determine whether each stationary point is a maximum or minimum point. (2)
- c State the set of values of  $k$  for which the equation
- $$2 + 9x + 3x^2 - x^3 = k$$
- has three solutions. (2)
- 10  $f(x) = 4x^3 + ax^2 - 12x + b$ .
- Given that  $a$  and  $b$  are constants and that when  $f(x)$  is divided by  $(x + 1)$  there is a remainder of 15,
- a find the value of  $(a + b)$ . (2)
- Given also that when  $f(x)$  is divided by  $(x - 2)$  there is a remainder of 42,
- b find the values of  $a$  and  $b$ , (3)
- c find the coordinates of the stationary points of the curve  $y = f(x)$ . (6)