The mean is the sum of all the data values divided by the number of values.

To calculate the mean, write the frequency table vertically and add a column to it.

Worked example 2

Find the mean number of goals for the data in Worked example 1.

Number of goals, x	Number of teams, f	f × x
0	1	1 × 0 = 0
1	3	3 × 1 = 3
2	6	6 × 2 = 12
3	2	2 × 3 = 6
4	4	4 × 4 = 16
5	3	3 × 5 = 15
6	2	2 × 6 = 12
Total	$\Sigma f = 21$	$\Sigma fx = 64$

Add a column for $f \times x$ and a row for totals.

Work out $f \times x$ for each row.

Sum the f and $f \times x$ columns.

6 teams scoring 2 goals each makes 12 goals in total.

$Mean = \dfrac{\Sigma fx}{\Sigma f} = \dfrac{64}{21} = 3.048$ (correct to 3 decimal places).

Work out $\dfrac{\Sigma fx}{\Sigma f}$

For discrete data in a frequency table, mean $= \bar{x} = \dfrac{\Sigma fx}{\Sigma f}$

Hint

$\bar{x}$ is the mean of all the values of x.

3 The frequency table shows the number of times students in a Year 11 class are late during a term.

Number of times late	Number of students	f × x
2	3	
3	12	
4	6	
5	4	
6	7	
7	2	
8	1	

Find the mean.

Q4 hint

You can enter the data into a table in a spreadsheet, calculate the $f \times x$ values and all the totals, and use these to calculate the mean.

4 During two consecutive months a gardener recorded the temperature, in degrees Celsius, at the same time each day. His results are shown in the table.

Temperature (°C)	18	19	20	21	22	23
Number of days	5	8	19	14	12	3

For these temperatures, find:

a the mode

b the median

c the mean.

This frequency table shows the ratings a random sample of 40 students gave a new app.

Rating A means they enjoyed it very much.
Rating E means they did not enjoy it at all.

Rating	A	B	C	D	E
Number of students	6	13	10	7	4

a Work out:

   i the mode   ii the median rating.

b Explain why you cannot calculate the mean rating.

The frequency table gives information about the number of GCSE subjects each student is taking.

Number of subjects	6	7	8	9	10	11
Number of students	12	15	20	29	19	7

For this data work out:

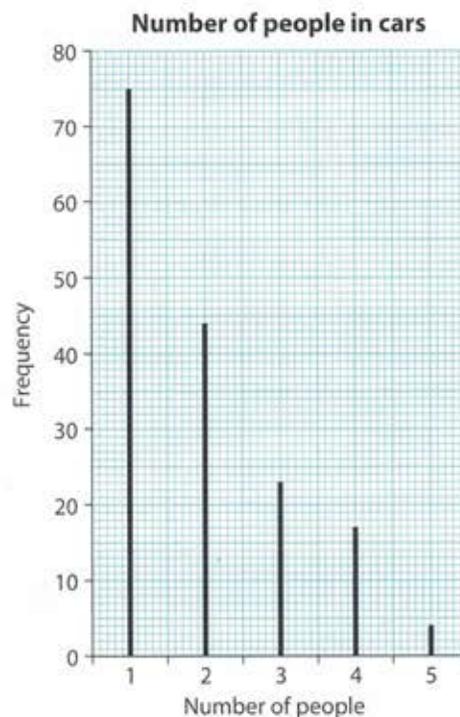a the mode   b the median   c the mean.

This vertical line graph shows the number of people in each car that passed the gates of a school between 9 am and 10 am.

a What is the mode?

b Display this data in a frequency table.

c Find the median.

d Find the mean.

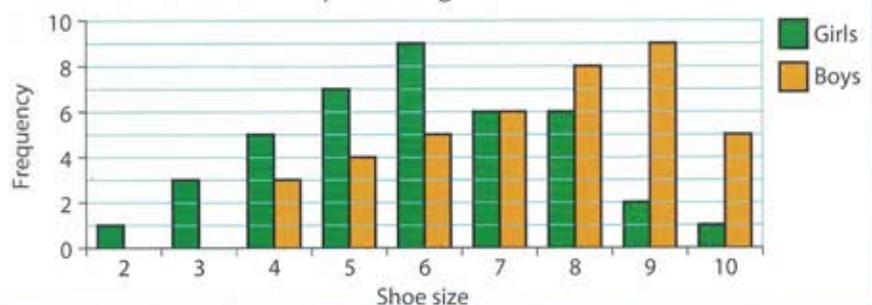**Number of people in cars**

Q7c hint

Use the frequency table.

Exam-style question

8 This multiple bar chart gives information about the shoe sizes of 40 boys and 40 girls.

For the boys' results:

a write down the mode   **(1 mark)**

b find the median   **(2 marks)**

c work out the mean.   **(2 marks)**

3.3 Averages from grouped data

Learning objectives

- Find the modal class and the class containing the median from grouped data.
- Calculate estimates of the mean and median from grouped data.
- **H** • Calculate estimates of the median and mean from grouped data with unequal class widths and histograms.

The table below shows information about the amount of time that some girls spent watching television in one week.

Number of hours, x	Number of girls, f
$0 \leqslant x \leqslant 5$	3
$5 < x \leqslant 10$	7
$10 < x \leqslant 15$	10
$15 < x \leqslant 20$	4

Each group is called a class interval. $5 < x \leqslant 10$ is a class interval. The class boundaries are 5 and 10. The class width is $10 - 5 = 5$.

Total number of girls $\Sigma f = 3 + 7 + 10 + 4 = 24$.

We cannot tell exactly how many hours each of the 24 girls spent watching television. The 3 who spent between 0 and 5 hours could have watched any amount between 0 and 5 hours.

So the mean, mode and median can only be estimated for this data.

Key point 1

The **modal class** is the class with the highest frequency.

The mode cannot be given for the data above but the modal class is $10 < x \leqslant 15$ as this is the class with the greatest number of girls in it.

Key point 2

For grouped continuous data, or for large data sets, the median is the $\frac{1}{2}$ nth value.

Worked example 1

This table shows information about the amount of time that some girls spent watching television in one week.

Number of hours, x	Number of girls, f
$0 \leqslant x \leqslant 5$	3
$5 < x \leqslant 10$	7
$10 < x \leqslant 15$	10
$15 < x \leqslant 20$	4

Find the class interval that contains the median.

Number of hours, x	Number of girls, f	Cumulative frequency
$0 \leqslant x \leqslant 5$	3	3
$5 < x \leqslant 10$	7	10
$10 < x \leqslant 15$	10	20
$15 < x \leqslant 20$	4	24

Add a cumulative frequency column to the table.

There are 24 girls, so the median is the $\frac{1}{2} \times 24 = $ 12th girl. The class interval $10 < x \leqslant 15$ contains the median result.

1 A random sample of leaves was collected from a tree. The lengths (x cm) of the leaves were recorded. They are shown in the table.

Length of leaves, x (cm)	Number of leaves, f
$4 \leqslant x \leqslant 6$	6
$6 < x \leqslant 8$	12
$8 < x \leqslant 10$	5

a What is the modal class?

b Which class interval contains the median?

2 The table shows 60 students' marks in their Art exam.

Mark	20–29	30–39	40–49	50–59	60–69	70–79	80–89	Total
Frequency	3	7	13	16	13	5	3	60

a Which class interval contains the median?

b Which is the modal class?

To calculate an estimate for the median, you need to assume that the data in the class containing the median is evenly spread across the class.

Hint

It is unlikely that the data is evenly spread. This is why the median you calculate is only an estimate.

Worked example 2

The table shows the number of hours 24 girls spent watching television in one week.

Number of hours, x	Number of girls, f	Cumulative frequency
$0 \leqslant x \leqslant 5$	3	3
$5 < x \leqslant 10$	7	10
$10 < x \leqslant 15$	10	20
$15 < x \leqslant 20$	4	24

10 girls watched for 10 hours or less.

20 girls watched for 15 hours or less.

Work out an estimate for the median amount of time spent watching television.

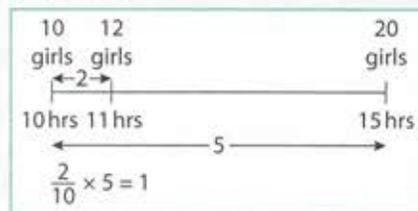The 12th girl is in the $10 < x \leqslant 15$ class interval.

This class interval is for the 11th to 20th girls, so the 12th girl is 2 girls in. There are 10 girls in the class interval.

There are $15 - 10 = 5$ hours between the class limits.

So, we need to go $\frac{2}{10}$ of 5 hours into this class.

$\frac{2}{10}$ of 5 hours = 1 hour so go 1 hour into this class.

The estimated median is
$10 + 1 = 11$ hours.


```
10      12                              20
girls   girls                          girls
       ←2→
10 hrs 11 hrs                          15 hrs
            ←————————— 5 —————————→
         2
        ── × 5 = 1
        10
```

This method is called **linear interpolation**. You can use the method shown in Worked example 2, or the formula in Key point 3.

Key point 3

Estimated median $= L + \dfrac{\frac{n}{2} - F}{f} \times w$, where:

- L is the lower boundary of the class containing the median
- n is the total number of values
- F is the cumulative frequency of the intervals before the one containing the median
- f is the frequency of the median class interval
- w is the width of the median class interval.

Using the formula with data from Worked example 2:

$L = 10$, $n = 24$, $F = 10$, $f = 10$, $w = 5$

Number of hours, x	Number of girls, f	Cumulative frequency
$0 \leqslant x \leqslant 5$	3	3
$5 < x \leqslant 10$	7	⑩
$10 < x \leqslant 15$	⑩	20
$15 < x \leqslant 20$	4	㉔

Estimated median $= L + \dfrac{\frac{n}{2} - F}{f} \times w$

$$= 10 + \dfrac{\frac{24}{2} - 10}{10} \times 5$$

$$= 10 + \frac{2}{10} \times 5$$

$$= 11$$

3 Calculate an estimate of the median for the data in question **1**.

4 Some people were asked to record the amount of time they spent watching television on one particular Saturday. The results are shown in the table.

Number of hours, x	Frequency, f
$0 \leqslant x \leqslant 3$	6
$3 < x \leqslant 6$	24
$6 < x \leqslant 9$	10

a What is the modal class?

b Work out an estimate for the median time.

To estimate the mean of grouped data, assume that in any class the data is evenly spaced about the midpoint of the class limits. The midpoint is found by adding the class limits together and dividing by 2.

Then use the midpoint in the same calculations as for data in an ungrouped frequency table.

Key point 4

An **estimated mean** can be found from a grouped set of data using the formula:

mean $= \dfrac{\Sigma(f \times \text{midpoint})}{\Sigma f}$, where Σ means 'the sum of' and f is frequency.

> **Hint**
>
> This is an estimate because the actual values of the data are unknown.

Worked example 3

The table shows information about the amount of time that some girls spent watching television in one week.

Calculate an estimate for the mean number of hours the girls spent watching television.

Number of hours, x	Number of girls, f
$0 \leqslant x \leqslant 5$	3
$5 < x \leqslant 10$	7
$10 < x \leqslant 15$	10
$15 < x \leqslant 20$	4

Number of hours, x	Midpoint	Number of girls, f	$f \times$ midpoint	
$0 \leqslant x \leqslant 5$	2.5	3	7.5	Add a midpoint column and a $f \times$ midpoint column.
$5 < x \leqslant 10$	7.5	7	52.5	
$10 < x \leqslant 15$	12.5	10	125.0	Sum the columns.
$15 < x \leqslant 20$	17.5	4	70.0	
	Total	24	255.0	Calculate the mean using the formula.

$Mean = \dfrac{\Sigma(f \times \text{midpoint})}{\Sigma f} = \dfrac{255}{24} = 10.625$ hours

5 The speeds of some cars on a motorway are given in the frequency table.

Speed, x (mph)	Number of cars
$20 < x \leqslant 30$	3
$30 < x \leqslant 40$	10
$40 < x \leqslant 50$	17
$50 < x \leqslant 60$	30
$60 < x \leqslant 70$	35
$70 < x \leqslant 80$	5

Use this information to work out:

a the modal group for the speed of the cars used in this survey

b an estimate for the median car speed

c an estimate for the mean car speed.

6 The table shows 60 people's scores in a computer game.

Work out:

a the modal class for the scores

b an estimate for the median score (to 2 dp)

c an estimate for the mean score (to 2 dp).

Score	Frequency
0–9	12
10–19	2
20–29	6
30–39	4
40–49	14
50–59	10
60–69	2
70–79	10
Total	60

Q6c hint

The midpoint of the 0–9 class is at $\dfrac{0 + 9}{2}$

7 The ages of some people watching a film are given in this frequency table.

Age, x (years)	Number of people
$10 \leqslant x < 20$	4
$20 \leqslant x < 30$	15
$30 \leqslant x < 40$	11
$40 \leqslant x < 50$	10

Work out:

a the modal class for the age of the people watching the film

b an estimate for the median age of the people watching the film

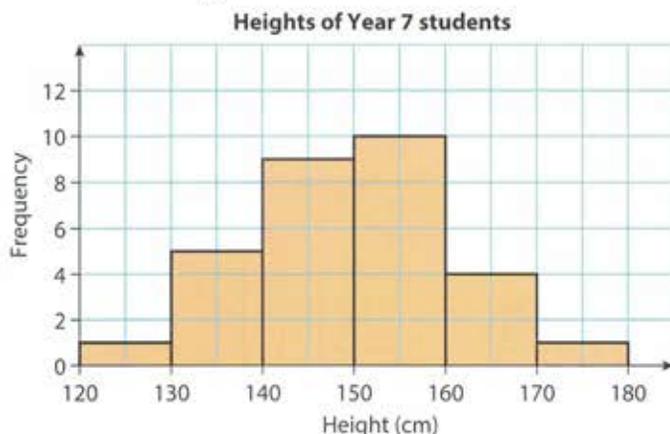**c** an estimate for the mean age of the people watching the film.

Q7 hint

You can enter the data in a spreadsheet and calculate the midpoints, Σf and $\Sigma(f \times \text{midpoint})$, and the mean $\dfrac{\Sigma(f \times \text{midpoint})}{\Sigma f}$.

8 The histogram shows the heights of Year 7 students.

Heights of Year 7 students

a What is the modal class interval?

b Represent this data in a grouped frequency table.

c Calculate an estimate for the median height.

d Calculate an estimate for the mean height.

Give your answers to a suitable degree of accuracy.

> **Q8c hint**
>
> Use the frequency table.

Exam-style question

9 The frequency table shows the lengths of leaves. The data has been grouped into classes of unequal widths.

Length, l (cm)	Frequency
$3 \leqslant l < 4$	3
$4 \leqslant l < 5$	4
$5 \leqslant l < 8$	14
$8 \leqslant l < 10$	20
$10 \leqslant l < 11$	9

a Calculate an estimate for the mean. **(3 marks)**

b Work out an estimate for the median. **(2 marks)**

Statistics - Revision Questions

Complete all questions carefully on the handout

1. Sixty people were asked to name their favourite season.
   The results are given below:

Season	Number of people
Spring	15
Summer	25
Autumn	16
Winter	4

   If the pie chart is to be drawn, calculate the angle required for the summer.

   i.   150°,

   ii. 15°,

   iii. 125°.

---

2. Ten historians estimate the age of an ancient plate.
   The results in years are:

   2200 2600 2300 2350 2450 2400 2200 2200 2450 2300

   Smart says that the mode of these values is 2200 years.
   Is Smart correct? Tick a box.

   ☐ Yes   ☐ No

   Give a reason for your answer.

---

3. Some friends are choosing what they would like to do on holidays. The table shows their choices.

Choice	Male	Female
Theme Park	12	3
Water Park	7	8
Sunbathing	5	11
Coach Trip	2	2
Total	26	

a) How many friends are there in total?

b) What fraction of the friends chooses sunbathing?

The pictogram shows the choices of the 26 males.

Theme Park	𝕩 𝕩 𝕩 𝕩 𝕩 𝕩
Water Park	𝕩 𝕩 𝕩 𝕩
Sunbathing	𝕩 𝕩 𝕩
Coach Trip	𝕩
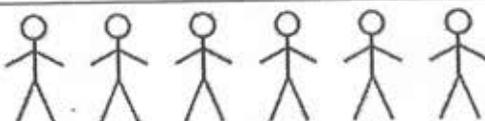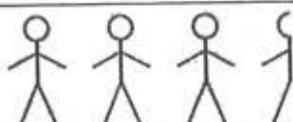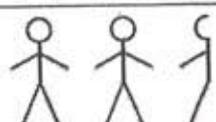
c) Complete the key: 𝕩 represents _____ friends.

8. Kate buys 20 boxes of drawing pins. She counts the number of drawing pins in each box. The table shows the information about her results.

Number of drawing pins	Frequency	$f \cdot x$	Cumulative frequency
28	4		
29	5		
30	3		
31	8		

a) Calculate the mean number of drawing pins in the 20 boxes.

b) Find the median number of drawing pins.

9. Fifty students are timed when running one kilometre.
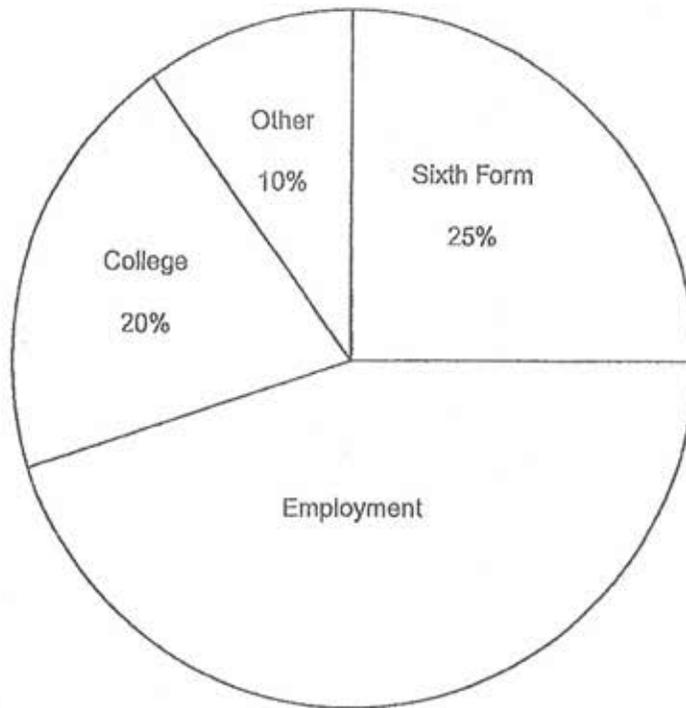The results are shown in the table.

Time (t minutes)	Frequency		
$4.0 < t \le 4.5$	2		
$4.5 < t \le 5.0$	7		
$5.0 < t \le 5.5$	8		
$5.5 < t \le 6.0$	18		
$6.0 < t \le 6.5$	10		
$6.5 < t \le 7.0$	5		
TOTAL:			

a) Write down the modal time interval.

b) Calculate an estimate of the mean time.

(c) Find an estimate for the median class

10 The pie chart shows the destinations of 300 students from Year 11 in 1979.

(a) Work out the percentage of the students who went into Employment.

Answer: _____%

(b) Work out the number of students who went to College.

Answer: _____students

4. The mean weight of five students is 68.3 Kg.
   If a new student is taken into consideration the mean becomes 70.5 Kg.
   Calculate the weight of the new student

5. A class of 18 boys and 12 girls took a Science exam.
   The mean mark for the boys is 62.5%.
   The mean mark for the girls is 58.75%.

   Work out the mean mark for the whole class.

6. Here are the marks scored in a Maths quiz by the students of a class.

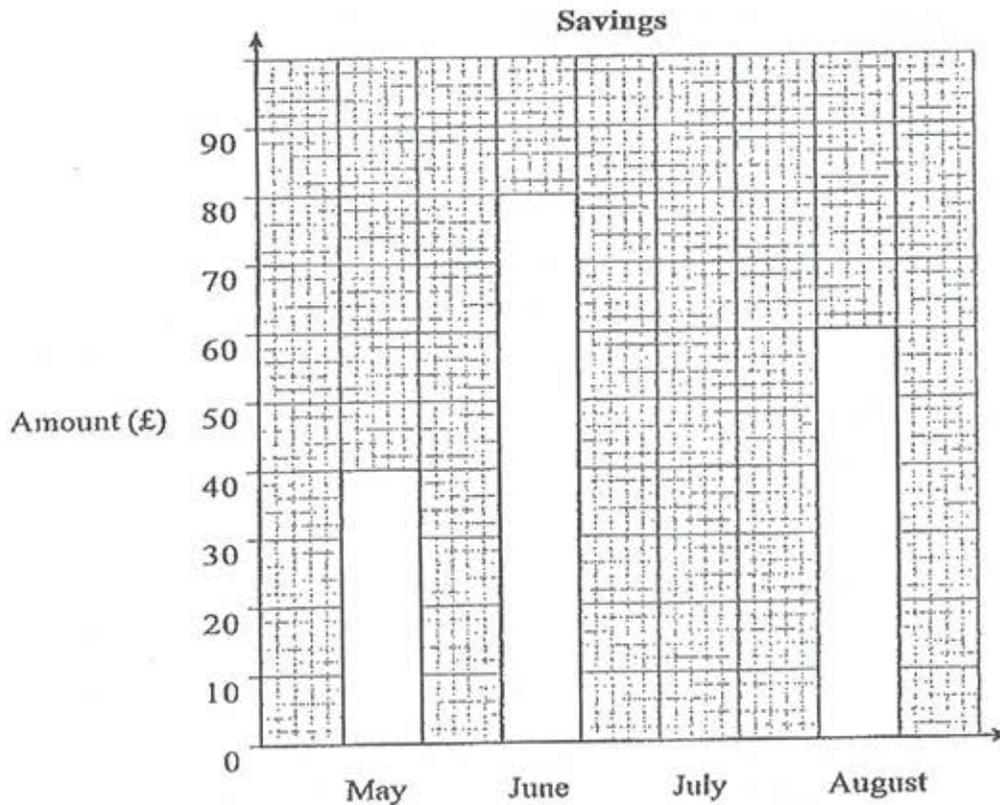   2, 13,15,16,4,6,19,10,11,4,5,15,4,16,6

   Calculate:
   a) The range,

   b) The median,

   c) The mean mark of the students, correct to 3 significant figures.

7

(a) The bar chart shows the amounts Isaac saves in May, June and August 2010.

Savings

i. How much does he save in May 2010?

Answer: £_____

ii. From May to August he saves £250 in total.
   Complete the bar chart by drawing the bar for July.
   Show your workings clearly.

(b) The pictogram shows the amounts Isaac saves in the next four months.